Wittgenstein's Aesthetics in the Philosophy of Mathematical Practice

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Abstract

The philosophy of mathematical practice has often seen Wittgenstein as one of its early "mavericks", later followed by Lakatos and others. However, most authors in the philosophy of mathematical practice never acknowledge his writings on aesthetics, and Wittgenstein himself does not seem to make a strong connection either.

This paper tries to fill this gap. I claim that sentences like "This theorem is *interesting*" or "This proof is *beautiful*" are aesthetic judgements in the Wittgensteinian sense: one judges according to the rules determining mathematical culture, canon and practice, and is trained to do so. Through this perspective, the paper shows that Wittgenstein's thought on aesthetics is an interesting point of view to understand mathematical culture and canon. In particular, I look at how Wittgenstein's aesthetics connect with Roy Wagner's constraints-based philosophy of mathematics, a framework for mathematical practice that already relies on Wittgenstein's philosophy but does not take aesthetic issues into account.

1 Introduction

Mathematics is often said to possess great beauty. Russell's famous quote illustrates this best:

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty —a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. (Russell 1907)

This seems to rely on a simultaneous belief in a Platonic idea of truth and beauty. Mathematics would be a form of absolute knowledge, and hence it could have absolute, "supreme" beauty.

However, when working mathematicians talk about a theorem being "enlightening" or a proof being "beautiful", it does not seem like they are pointing at the same type of supreme beauty Russell is talking about. It is more about how these theorems and proofs fit into the greater picture of their research programmes, mathematical culture and canon, something more mundane, more down-to-earth.

If somebody in the philosophy of mathematics disagrees with this idea of absolute supreme beauty is likely to be the late Wittgenstein, whose thoughts on aesthetics point to a more conventionalist and relativist standpoint. In parallel to Wittgenstein, the academic philosophers most keen to criticise the previous quote as contributing to obscuring and mystifying what mathematics actually is are the philosophers of mathematical practice. In this sense, it is no coincidence that the latter have often used Wittgenstein's philosophy of mathematics in their work to support the idea that mathematics is nothing other than what real mathematicians do, the language game that is played in practice, and not some idealised picture of it, rigourously formalised in the predicate calculus.

Interestingly, although Wittgenstein's late work on mathematics and epistemology is an important source of inspiration for the philosophers of mathematical practice, they have, for the most part, ignored the aesthetic dimension of Wittgenstein. His work related to such topics is never quoted. The issue of aesthetic appreciation in mathematics is rarely brought up, perhaps because quotes like that one by Russell above are perceived as nothing other than distracting verbiage.

In this paper, I contend that the philosophy of mathematical practice can significantly benefit from looking at Wittgenstein's aesthetics, mainly as described in his 1938 *Lectures on Aesthetics* and some of the passages in *Culture and Value*. I suggest that phrases like "This theorem is interesting" or "This proof is beautiful", when analysed under the lens of Wittgenstein, acquire new layers of meaning that fit tightly into existing work in mathematical practice and constitute an interesting point of view to understand mathematical culture and canon.

The structure of the paper is as follows. Section 2 starts by introducing the general motivation of the philosophy of mathematical practice and how Wittgenstein's ideas have often been used in this context. Section 3 observes that, despite the previous connections, Wittgenstein's aesthetics are rarely used in mathematical practice. As a starting point, I review the basic ideas held by Wittgenstein in the light of two passages from *Culture and Value* relating mathematics and aesthetics. Section 4 pushes this connection further by looking at a specific philosopher of mathematical practice, Roy Wagner. I present his constraint-based philosophy of mathematics and briefly discuss how the point of view of aesthetics contributes to two of his main ideas: dismotivation and consensus in mathematics. Section 5 provides a conclusion and points to possible further lines of research.

2 Wittgenstein and the philosophy of mathematical practice

Despite Wittgenstein's claim that his "chief contribution has been in the philosophy of mathematics" (Monk 1990, 466), the reception of his ideas in this area has traditionally been mixed at best. For his contemporaries, such as Dummett (1959) and Kreisel (1958), Wittgenstein's "full-blooded conventionalism" (Dummett 1959) about mathematics was to be rejected, in part due to technical errors in his discussion of higher mathematics and Gödel's incompleteness results, in part due to his strong anti-foundational character. These initial reactions, combined with the posthumous publication of his works, coming from disjointed notes and disconnected lectures written in his famously heterodox style, made approaching Wittgenstein's philosophy of mathematics challenging.

Time has seemed to rectify this early judgment, with later commentators seeing him as the "turning point" in the philosophy of mathematics (Shanker 2013). His thought has gradually gained approval, especially from a sector of academic philosophy somewhat discontent with the traditional philosophy of mathematics: the philosophers of mathematical practice.

Tired of endless scholarly debates about access to abstracta and technical quarrels regarding logic and foundational schools, the philosophy of mathematical practice tries to look at what mathematicians actually do. As Corfield writes in one of the earlier works in the field,

> researchers here believe that a philosophy of mathematics should concern itself with what leading mathematicians of their day have achieved, how their styles of reasoning evolve, how they justify the course along which they steer their programmes, what constitutes obstacles to their programmes, how they come to view a domain worthy of study and how their ideas shape and are shaped by the concerns of physicists and other scientists. (Corfield 2003, 10)

Wagner phrases a similar goal:

> Instead of asking foundational questions about the grounding of mathematics, its freedom, its unique position, its monsters, or its source of authority, we will ask some questions about mathematical practice. How are mathematical statements used? How do people get to agree on them? How are they interpreted? (Wagner 2017, 59)

Similar points of view are expressed by Mancosu (2008), Aspray and Kitcher (1988), and Lakatos (1976), who, in his influential *Proofs and Refutations*, advocates for a focus on the actual mathematical practice by looking at its historical development in the hands of its practicioners.

The philosophers of mathematical practice are trying to study actual mathematical cultures rather than discuss an idealised picture of what mathematics should be. This fits Wittgenstein quite well, who would be more worried about the different language games that constitute mathematics than about abstract philosophical problems that seem to be nothing more than linguistic misunderstandings[1]. More particularly, Wittgenstein seems to share three fundamental points with the philosophers of mathematical practice[2]. Firstly, a remarkable anti-foundationalist standpoint (stronger than simple pluralism about foundations). Secondly, the belief that mathematics can live with contradictions, moving past the obsession with consistency found in mathematical logic and analytic philosophy. Thirdly, the stance mathematical propositions are better interpreted as standards than as descriptions. In short, it is more inter-

1. Corfield's claim that "the philosophy of mathematical practice is the only real philosophy of mathematics" (Corfield 2003) is reminiscent of Wittgenstein's rejection of the traditional ways of doing philosophy, which produced him disgust: "I am in a sense making propaganda for one style of thinking as opposed to another. I am honestly disgusted with the other." (Wittgenstein 1966, 23)

2. See here (Wagner 2020) for a summary of Wittgenstein's philosophy of mathematics as understood by the philosophy of mathematical practice.

esting to discuss how people *use* mathematical propositions than to discuss whether they are referring to real mathematical objects living in some outer Platonic realm.

Traditionally, philosophers of mathematical practice have tended to see Imre Lakatos and his seminal work in the late 60s and early 70s as the earliest of their "mavericks", but this perception seems to be changing. On the one hand, as Pérez-Escobar (2022) points out, Wittgenstein can be argued to predate Lakatos on many fronts[3], and many contemporary philosophers of mathematical practice bring up Wittgenstein's work in some of their chief arguments. See, for example, Sørensen (2009), on the transition from "formula-centered" to "concept-centered" mathematics, or Pérez-Escobar and Sarikaya (2022), on the dichotomy between pure and applied mathematics. In this paper, we shall mainly focus on the use of these ideas in the work of Roy Wagner, whose constraint-based philosophy of mathematics owes a lot to Wittgenstein. We come back to his philosophical system in Section 4.

In short, despite an early mixed reception, Wittgenstein's work on the philosophy of mathematics has found a home in the emerging field of mathematical practice. However, as I discuss next, the scope of this interest might be a bit too narrow.

3 Wittgenstein's aesthetics

In his *Remarks on the Foundations of Mathematics*, Wittgenstein uses the word "aesthetics" a total of four times in over 300 pages, never with much depth. In his 1939 *Lectures on the Foundations of Mathematics*, the word appears barely twice, in the same sentence, briefly mentioning the alleged aesthetic pleasure derived from mathematics, an issue never to be addressed again.

Perhaps due to this explicit absence of aesthetic issues in his writings about mathematics, both Wittgenstein's more mathematically-focused commentators (except, perhaps, for Säätelä (2013)) and the philosophers of mathematical practice have mostly ignored his thought on aesthetics[4]. This is surprising on two fronts. On one side, Wittgenstein himself seems to have been quite interested in aesthetic experience, judging by his *Lectures on Aesthetics* and some others of his writings and the interest in them in the secondary literature. On the more contemporary side, the philosophers of mathematical practice are surely interested in how sentences like "This theorem is *interesting*" or "This proof is *beautiful*" are used by real mathematicians. Since, for them, mathematics is about the real thing and not about abstract truths floating beyond our reality, the way such aesthetic judgments are uttered can likely tell us a lot about how mathematical communities operate and produce new results.

In order to look at this connection more closely, we can start by looking at what Wittgenstein himself has to say. In his canonical mathematical writings, he does not mention this issue but he does make some connections in some of the notes collected in *Culture and Value*. At the very beginning, he notes that "There is no religious denomination in which so much sin has been

3. Though, interestingly, it does not seem like Lakatos acknowledges much influence from Wittgenstein. Pérez-Escobar brings up the anecdote of how Lakatos claimed that "Wittgenstein was the biggest philosophical fraud of the twentieth century" (Pérez-Escobar 2022, 9), despite some early enthusiasm about his work.

4. For that matter, Pérez-Escobar, who is the author that has addressed the connection between mathematical practice and Wittgenstein most explicitly in (Pérez-Escobar 2022; Pérez-Escobar and Sarikaya 2022), never mentions aesthetics.

committed through the misuse of metaphorical expressions as in mathematics" (Wittgenstein 1998, 15). Though the relation between mathematics and religious belief would be an exciting subject in its own right, we shall focus only on aesthetics for this time. For that matter, two quotes from *Culture and Value* seem to be key:

> The queer resemblance between a philosophical investigation (perhaps especially in mathematics) and one in aesthetics. (e.g., what is bad about this garment, how it should be, etc..) (44)

> That writers, who after all were something, go out of date is connected with the fact that their writings, when complemented by the setting of their own age, speak strongly to people, but that they die without this complementation, as if bereft of the lighting that gave them colour. And I believe that the beauty of mathematical demonstrations, as experienced by Pascal too, is connected with this. Within this way of looking at the world, these demonstrations did have beauty—not what superficial people call beauty. A crystal too is not beautiful in every "setting"—though perhaps everywhere attractive. (109)

In the first quote, Wittgenstein addresses some resemblance between aesthetic investigations and philosophical investigations about mathematics, though not necessarily between mathematics and aesthetics themselves. On the second, he addresses the issue of when a proof is considered beautiful and ties this to the specifics of a culture.

Let us have a brief look at Wittgenstein's thoughts on aesthetics in the light of these two quotes. For Wittgenstein, aesthetics is yet another language game. In this game, one utters aesthetic judgements or appreciations. Like in any language game, these judgements follow some normative rules. In order to successfully play the game, one needs knowledge and expertise, which are attained through some form of training and education. Crucially, the object for aesthetic appreciation can be anything, not only high art. Wittgenstein often thinks of different forms of craftsmanship as aesthetic objects (garments, furniture...) and, for that matter, one could also include mathematics.

As Reguera (1992) points out, we can imagine Wittgenstein's aesthetics as the intermediate layer between a practice and its philosophy, where each layer tries to make sense of the one below. If we take the subject matter of aesthetics to be the practice of art, then aesthetics would be the object language that models this practice, with linguistic norms dictating how this practice is to be judged: what is considered nice and what is ugly, what is correct and what is not. On a third level, there would be the philosophical investigation on aesthetics, which would be the meta language describing the object language.

As Wittgenstein points out, "to describe their use [of aesthetic expressions], you have to describe a culture" (Wittgenstein 1966, 8, Lecture I.25). Every culture presents different rules for the language game of aesthetics. Aesthetic judgments only gain meaning in some context where they can be used. Once such a context is fixed, the language game proceeds by making aesthetic judgments and producing aesthetic explanations. These are characterized by some form of (i) consensus, (ii) persuasion and (iii) case analysis and comparison (Reguera 1992, 22).

Under this framework, mathematics fits as yet another language game, or at least as a family of language games. Some of them are likely to behave like aesthetic games, and in order to understand such a game one needs to describe cultures. This is precisely what happens with mathematical proofs, as described in Wittgenstein's second quote above. After all, would Euler have found Gödel's theorems interesting or insightful? Their proofs ingenious or beautiful? Or even correct? Definitely not. In Euler's mathematical culture, what was considered mathematics proper (i.e., the set of mathematical objects worthy of aesthetic appreciation or disdain) was very different from the mathematics of our age.

To describe and explain mathematical cultures is precisely what the philosophy of mathematical practice is set out to do. This ties in with the first one of the quotes above: "The queer resemblance between a philosophical investigation (perhaps especially in mathematics) and one in aesthetics".

As Säätelä (2013) points out, it is not that the subject matter of mathematics and aesthetics is the same (say, that painting and deriving theorems are the same thing). Not at all: it is about how the intermediate systems organizing these practices resemble each other. How the way we make sense of the object language describing the practice of art is similar to how we make sense of mathematical practice by unveiling the hidden rules of mathematical cultures.

This explains why looking at sentences like "This proof is beautiful" from the optic of Wittgenstein's aesthetics fits so well in the philosophy of mathematical practice: to understand what "beautiful" means in that context, one needs to describe mathematical culture, mathematical canon. To perform such an utterance, one needs to have been trained in a particular mathematical style that teaches one to appreciate the relevance of such an argument. It is not about some abstract notion of beauty applied to the proofs coming from The Book; it is about how a proposition or a proof is *used* and what it tells us about the rules involved in the language game.

4 Wittgenstein's aesthetics in Roy Wagner's philosophy

As a particular case study into how philosophers of mathematical practice are using Wittgenstein and how his thoughts on aesthetics can fit into their picture, we will look into the theoretical framework of Roy Wagner, a contemporary philosopher of mathematical practice. His system, the so-called "constraint-based philosophy of mathematics", thinks of mathematics as a human practice emerging from a varied set of constraints, both sociological and cognitive. In his own words,

> instead of debating the reality of mathematical entities, we will think of mathematics as a field of knowledge that negotiates various kinds of real constraints. This contingent array of constraints and the various ways of juggling them lead to the formation of various different mathematical cultures. (Wagner 2017, 59)

Under this framework, Wagner presents mathematics as a practice happening in the open-ended process of reinterpretation around different semiotic systems; an epistemic practice lying in the realm of the analytic a posteriori, characterized by (i) *dismotivation* from empirical grounds and (ii) the potential for formalization as the ultimate mechanism for arbitration and *consensus*.

The reader is referred to Chapter 3 in Wagner's *Making and Breaking Mathematical Sense* (2017) for a complete introduction to this framework. Here, I focus on the specific ideas of dismotivation and consensus since they both rely on Wittgenstein's philosophy of mathematics. Though Wagner does not bring aesthetics into the picture, we will review these notions and see how aesthetics connects with them.

4.1 Dismotivation

Dismotivation is the process "indicating the gradual loss of a mathematical statement's empirical motivations and grounding" (Wagner 2017, 60). Take the example of arithmetic. First, arithmetic is made to suit experience. It comes from the experience of counting and adding things in real-life contexts, from calculations that are empirically carried out. Then, at some point, arithmetic starts to lose its empirical grounding: if we know there are 20 chickens and 35 goats but counting them in real life gives us 60, then we assume we must have miscounted: arithmetic becomes a standard instead of a description, a standard against which one measures reality. Then, once empirical grounding is lost, research questions arise intra-theoretically, disjointed from the original context in which these questions were relevant. As Wittgenstein writes, "all the calculi in mathematics have been invented to suit experience and then made independent of experience" (Wittgenstein 1989, 43).

In this way, "arithmetic rules are a point of reference independent of experience" (Wagner 2017, 62). And this is not too different from aesthetic rules! Those too start by collecting the tastes, needs and wishes of some group of people and are soon dismotivated and assimilated into a more abstract notion of beauty or correctness. Wittgenstein writes:

> You could regard the rules laid down for the measurement of a coat as an expression of what certain people want. People separated on the point of what a coat should measure: there were some who didn't care if it was broad or narrow, etc.; there were others who cared an enormous lot. The rules of harmony, you can say, expressed the way people wanted chords to follow their wishes crystallized in these rules (the word 'wishes' is much too vague.)! All the greatest composers wrote in accordance with them. (Wittgenstein 1966, 5, Lecture I.16)

At first, there might have been good practical reasons for the style of specific garments. A coat to be used in certain weather conditions or by certain social class is likely to have such and such characteristics, and that might have established some initial standards. Nevertheless, these soon lose their raison d'être, and fashion evolves on its own, without the need for empirical grounding:

> Take the case of fashions. How does a fashion come about? Say, we wear lapels broader than last year. Does this mean that the tailors like them better broader? No, not necessarily. He cuts it like this and this year he makes it broader. Perhaps this year he finds it too narrow and makes it wider. (13, Lecture II.8)

In short, Wittgenstein's philosophy of mathematics and aesthetics seem to agree on this crucial point: aesthetic (respectively mathematical) propositions are not descriptions but standards against which to judge correctness with respect to some rules. In this way, the questions of which mathematical propositions are considered "interesting" or "insightful" fits nicely into Wagner's idea of dismotivation, since both mathematical and aesthetic language games involve a great deal of dismotivation in their rules.

4.2 Consensus

Mathematics seems to achieve something that most other forms of knowledge cannot: a great deal of consensus. Working mathematicians rarely doubt the correctness of proofs published in peer-reviewed journals. Sometimes mistakes are found, for sure, and the ongoing push for computer-aided proof verification is a sign that mathematicians value correctness. However, the reason they care so much about it is in part because they genuinely think that total correctness is achievable in mathematics. They think so because, as practice shows, mathematicians quickly arrive at a consensus over their results, so correctness must be achievable.

According to Wagner, consensus in mathematics relies on the rigidity of the rules employed and their potential for formalization, which can be used as the final arbitration mechanism. This combination of factors seems to explain why mathematics looks concerned with some abstract and transcendental notion of truth:

> This state of affairs seems to lend plausibility to the claim that mathematics represents a deeper truth, or stands closer to truth than other branches of knowledge. How else did it happen that mathematical arguments are, unlike those of other sciences, reducible to consensually verifiable formalizations? But the exceptional truth of mathematics may be put into question if we consider the historical possibility that what guarantees consensus in mathematics is the active exclusion of arguments and concerns for which normal arbitration mechanisms are of no use. Mathematical consensus could arguably be the artifact of trimming mathematics along the dotted lines that allow consensus to be retained, rather than the expression of some essential mathematical trait. (Wagner 2017, 68)

But mathematical consensus goes beyond which proofs are accepted! Before a proof is even written, the idea for a theorem has to exist. For this to happen, somebody must have researched a topic well enough. Before that, a veteran faculty member must have approved funding for such a project. And, crucially, the criteria used to make such choices are essentially aesthetic in a Wittgensteinian sense: the criteria involve knowing what branches of mathematics are considered "relevant", "interesting", "beautiful", "insightful", "promising", "nice", and so on. Of course, the people in charge of making these aesthetic judgments have been trained according to the specific rules of specific mathematical cultures and are proficient in following them. This, in turn, trains younger researchers on what ideas, results, proofs and lines of inquiry are considered beautiful. After all, learning the rules for what can be appreciated as beautiful also implies learning the rules that dictate what is "ugly", "dirty", "messy", "convoluted", "cumbersome", "lacking intuition", "too technical", or even "trivial".

Hence, consensus in mathematics is possible because early aesthetic judgments that steer research programmes are made according to very rigid rules: aesthetic rules.

5 Conclusion

In this paper I have pointed to the connection between Wittgenstein's aesthetics and the ongoing work in the philosophy of mathematical practice. This emerging area has gradually adopted many of Wittgenstein's ideas (e.g., his focus on the practice of mathematics as language games, his strong anti-foundationalist stance, or his belief that mathematical propositions are standards rather than descriptions, amongst others), with some authors beginning to see him at the founding figure of the field, even predating Lakatos.

I have noted the disconnect between Wittgenstein's work on mathematics and the rest of his later philosophy, particularly regarding aesthetics and religious belief. Despite the seeming lack of interest both in Wittgenstein's published work and in the existing literature on mathematical practice to make this connection, I have claimed that the combination of these ideas is both possible and desirable. I have done so based on some of Wittgenstein's remarks from *Culture and Value*, giving an account of his aesthetics that can be made compatible with the central tenets of the philosophy of mathematical practice.

As a particular case study, I have looked at Roy Wagner's constraint-based philosophical framework. I briefly pointed at how two of the chief characteristics of mathematics under his philosophical system (dismotivation and consensus) are fleshed out even further by adding the lens of Wittgensteinian aesthetics, which fits tightly into the picture.

Two interesting questions remain unanswered. First, why do Wittgenstein's aesthetics fit so well in this context? I would like to claim that this might be due to the fact that Wittgenstein's thoughts on aesthetics are related to the epistemological framework laid out in some of his later work, especially in *On Certainty*. It would be interesting to study how much of *On Certainty* has influenced Wagner's work, who also thinks of external (physical, cognitive) constraints as forcing different frameworks of certainties that shape (and are shaped by) different practices or language games. Secondly, as pointed out in the opening lines of *Culture and Value*, Wittgenstein's thoughts on religious belief could also be an interesting addition to the philosophical toolbox of the philosopher of mathematical practice, an issue worthy of more attention.

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